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Experiments on transition of Mach reflexion

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Detailed experimental data are presented on the transition between regular and Mach reflexion. Data have been obtained for steady, pseudo-steady and unsteady flows, and include a study of the continuous and discontinuous transitions predicted by previous researchers. It is found that the criterion often used to calculate the transition condition is wrong in every case that we have investigated. In its place we propose an alternative criterion which has the property that the system remains always in mechanical equilibrium during transition.

1. Introduction

It is well known that the reflexion of a plane shock at a rigid wall will be a regular reflexion *RR* (sometimes also called a simple reflexion) if its angle of incidence ω_0 (see figure 1) is comparatively small, but that it will be a more complicated irregular or Mach reflexion *MR* for larger values of ω_0. The criterion commonly used to predict the transition $RR \leftrightarrow MR$ between the two wave systems makes use of the boundary condition that the flow downstream of the reflexion must be parallel to the wall. When applied to a regular reflexion this is interpreted as meaning that the streamline deflexion angle δ_1 through the reflected shock is equal in magnitude but opposite in sign to the deflexion angle δ_0 through the incident shock, hence

$$\delta_0 + \delta_1 = 0. \tag{1}$$

Equation (1) is violated when ω_0 increases to the point where it forces δ_0 to exceed in magnitude the shock detachment value of δ_1, so that

$$|\delta_0| \geqslant |\delta_{1\mathrm{det}}|. \tag{2}$$

The equality condition at detachment in (2) is often accepted as the criterion for the onset of Mach reflexion $RR \rightarrow MR$, and conversely for the inverse transition $RR \leftarrow MR$ (Landau & Lifshitz 1959, p. 412).

Experiments by Lean (1946) under steady-state conditions in a wind tunnel gave no reason to doubt the validity of (2). His work was done at free-stream Mach numbers M_0 of about 1·4 and 2·6. Other data for $M_0 \approx 2$ may be gleaned from a paper by Bardsley & Mair (1951). Although their interest was in the shock/boundary-layer interaction it is possible to obtain an estimate of the conditions for the transition $RR \leftrightarrow MR$ from their results, and again there is no reason to doubt (2). However, none of this work included a detailed study of transition. Doubts do begin to arise when one inspects the data obtained from shock-tube experiments; for example Bleakney & Taub (1949), working with data obtained by Smith (1945) from the diffraction of plane shock waves over corners, found that regular reflexion sometimes persisted well beyond the limits determined by (2). Other data obtained by Griffith & Bleakney (1954), Kawamura & Saito (1956) and by others amply confirm this conclusion. These experiments were conducted with unsteady shocks and revealed the useful fact that the wave systems were self-similar, which made it possible to remove the time t from the equation of motion (Jones, Martin & Thornhill 1951; Sternberg 1959). Such flows are often called pseudo-steady and it is valid to apply steady-state theory to a small neighbourhood about any point in a flow of this type and thus in particular to calculate local wave angles for comparison with experiment. The steady-state theory of regular reflexions has been given by Bleakney & Taub and by Stanyukovich (1960) and one can use it to calculate the wave angles at the shock confluence (reflexion) point. For Mach reflexion the wave angles at the confluence (triple) point can be calculated from the theory developed by Eggink (1943), Guderley (1947), Wuest (1948), Wecken (1949), von Neumann (1963), Henderson (1964) and Mölder (1971).

The object of the present paper is to report the results of a detailed experimental examination of the transition. This was done in an attempt to explain the apparent discrepancy between the wind-tunnel and shock-tube results. The models that we used are shown in figure 1. During the course of our work we found that (2) was wrong for every flow we studied. In its place we shall propose an alternative criterion which explains all the known experimental facts when the specific-heat ratio $\gamma = \frac{7}{5}$ and $M_0 \geqslant 2{\cdot}40$; these conditions correspond to there being either sonic or supersonic flow downstream of the reflected shock: $M_2 \geqslant 1$. For the range $1{\cdot}48 \leqslant M_0 \leqslant 2{\cdot}40$ the criterion is partly successful, in that it seems to predict where the experimental data will begin to deviate from the RR theory. In this case $M_2 < 1$, and in the limit $M_0 = 1{\cdot}48$ the criterion has a Mach-line degeneracy, that is the wave angle becomes equal to the Mach angle: $\omega_0 = \mu_0$. The criterion is of no value for $M_0 < 1{\cdot}48$.

2. Theory of the transition

In a paper of signal importance Kawamura & Saito have shown that the pressure downstream of the reflected shock can change either continuously or discontinuously during transition if (2) is accepted as the criterion. These two processes are illustrated by the polar diagrams in figure 2. It is found that for a perfect gas with $\gamma = \frac{7}{5}$ there will be a discontinuous transition when† $M_0 > 2{\cdot}23$ and there will be a continuous transition when $M_0 < 2{\cdot}23$.‡ Kawamura & Saito suggested that the continuous process was caused by the streamlines forming a sink on the ordinate of the polar diagram during transition and that this condition

† The English language version of Kawamura & Saito's paper gives $M_0 = 3{\cdot}203$ but this is thought to be a misprint.

‡ We shall continue to make the distinction between $M_0 = 2{\cdot}40$, where $M_2 = 1$, and $M_0 = 2{\cdot}23$, which is the boundary between continuous and discontinuous transition. Practically this difference in M_0 is too small to be resolved by experiment.

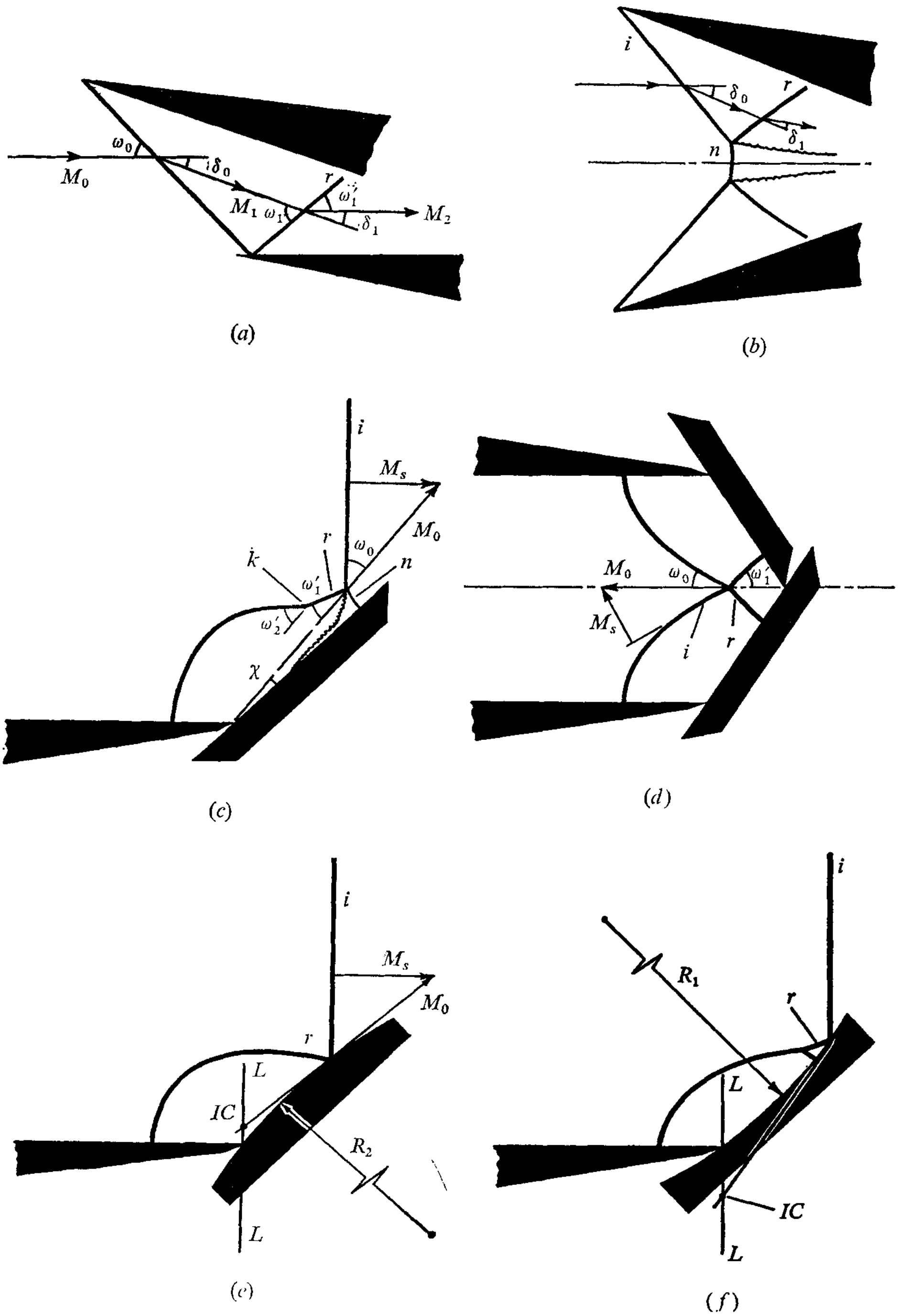
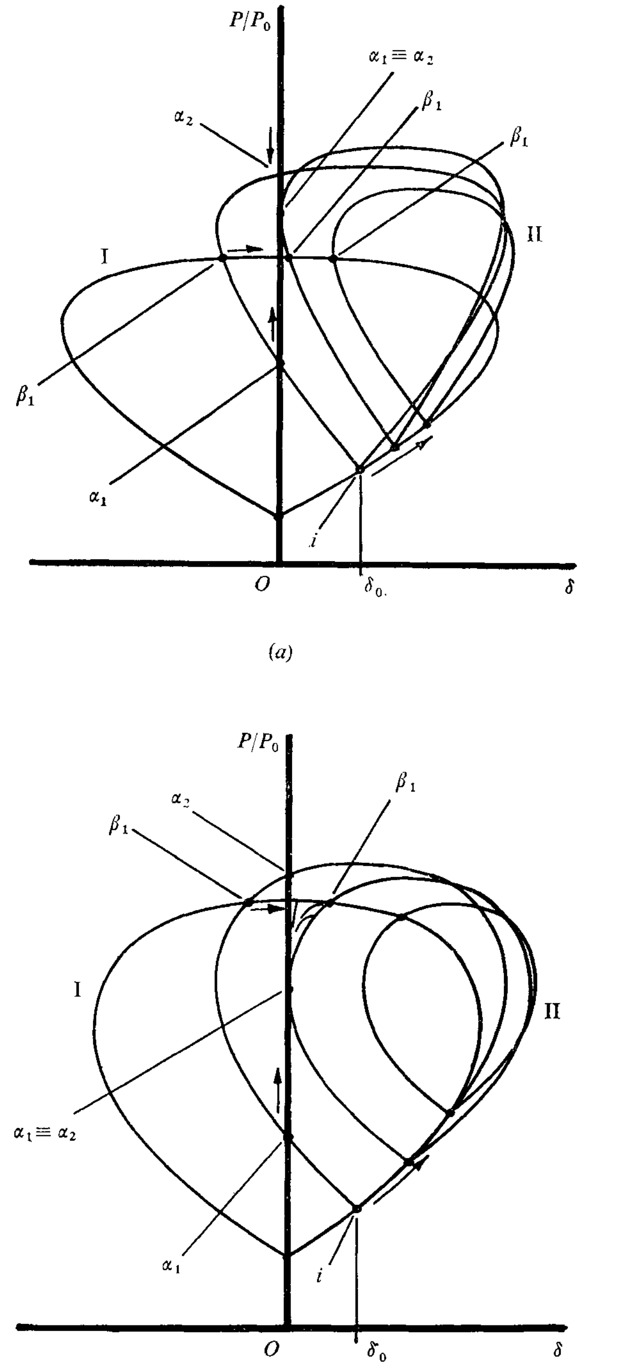


FIGURE 1. Models used to investigate transition to Mach reflexion. (*a*) Reflexion off a rigid wall in a wind tunnel, with minimal boundary-layer interaction. (*b*) Reflexion between two similar shock waves in a wind tunnel. (*c*) Reflexion off a plane rigid wall in a shock tube; *k*, kink in reflected shock. (*d*) Reflexion between two similar shock waves in a shock tube. (*e*) Reflexion off a convex rigid wall in a shock tube; *IC*, instantaneous corner; *LL*, locus of instantaneous corners; R_1, radius of curvature = 0·2 m. (*f*) Reflexion off a concave rigid wall in a shock tube; R_2, radius of wall curvature = 0·15 m.



(b)

FIGURE 2. Hodograph diagrams showing shock polar intersections at transition to Mach reflexion, after Kawamura & Saito. $\alpha_1 \equiv \alpha_2$, criterion (2). (*a*) $M_0 > 2{\cdot}40$. (*b*) $1{\cdot}48 < M_0 > 2{\cdot}40$; $\alpha_1 \equiv \alpha_2$ represents a sink on the ordinate.

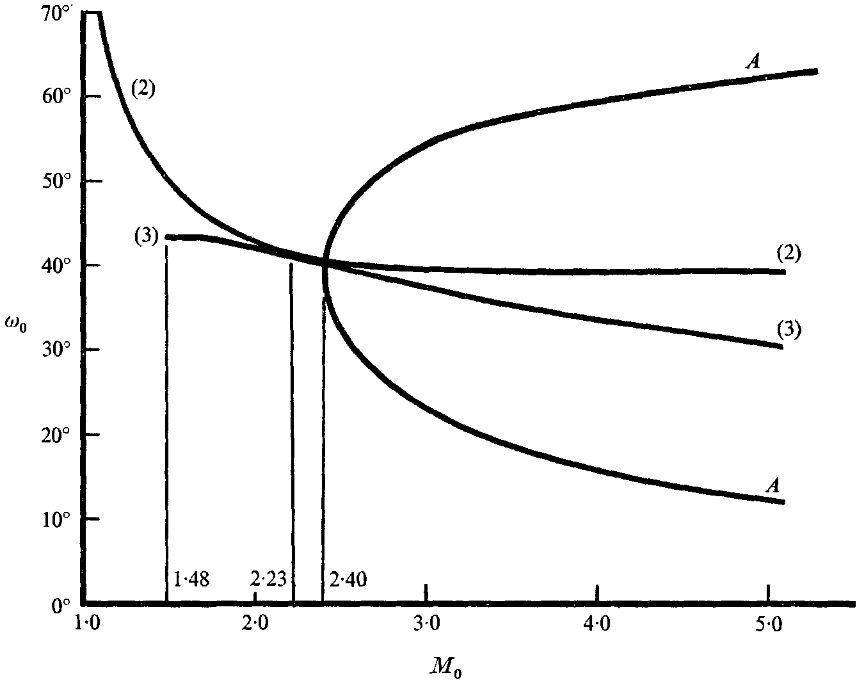


FIGURE 3. Criteria for the onset of Mach reflexion. (2), equation (2), $|\delta_0| = |\delta_{1\,\mathrm{det}}|$; (3), equation (3), $\delta_0 + \delta_1 = 0 = \delta_m$; A, $M_2 = 1$.

dominated the downstream flow. The significance of $M_0 = 2{\cdot}40$ is that it corresponds to $M_2 = 1$ for Mach reflexion, while $M_2 > 1$ when $M_0 > 2{\cdot}40$ and $M_2 < 1$ when $M_0 < 2{\cdot}40$. The $M_2 = 1$ condition is plotted in figure 3, which is replotted from a paper by Henderson (1965). The importance of this has been enhanced by the discovery that a slope discontinuity develops in the reflected wave for Mach reflexion, when $M_2 > 1$. Apparently this 'kink', as it is often called, is caused by the gas being deflected towards the wall by the *MR* system and then subsequently being brought back parallel to the wall by a band of compression waves. The situation is analogous to the steady-state reflexion of a shock wave by a flat plate in the presence of a boundary layer (Bardsley & Mair 1951; Holder & Gadd 1955, chap. 8; Chapman, Keuhn & Larson (1958). In any event, as M_0 increases the band of compression waves concentrates itself into a shock which has the appearance of being a second Mach stem, and the system is then sometimes called a double Mach reflexion. The kink was first noticed by White (1952) and since then it has been extensively studied by Merritt (1968), Weynantes (1968), Gvozdeava *et al.* (1969, 1970), Semenov, Syshchikova & Berezkina (1970) and Law & Glass (1971). With the help of pressure transducers both Merritt and Gvozdeava have shown that there is a steep positive pressure gradient downstream of the primary Mach stem and that this is associated with the formation of the secondary Mach stem. All of these experiments were done in shock tubes, mostly by using models similar to that shown in figure 1(c), but there is also at least one complementary series of wind-tunnel experiments on the phenomenon due to Lozzi (1971), who used a model similar to that shown in figure 1 (*b*). These last experiments revealed a sequence of up to four Mach reflexions, or combinations of regular and Mach reflexions.

Now a system which develops a pressure discontinuity during transition cannot be in mechanical equilibrium, where we define mechanical equilibrium to mean that the vector sum of the pressure forces and the momentum fluxes acting on the system is zero. If a pressure discontinuity occurs during transition then an unsteady wave of finite amplitude or a finite amplitude band of waves will be generated in the flow. These would be expansions for $RR \rightarrow MR$ and compressions for $RR \leftarrow MR$, but to the best of our knowledge they have never been reported. We therefore sought an alternative to (2) that would still satisfy (1) yet enable the system to remain always in mechanical equilibrium during transition. A suitable criterion can be constructed from the theory discussed by Mölder (1971). He takes into account the effect of shock-wave curvature at the confluence point and finds that the pressure in the downstream flow is then continuous during transition. It is readily concluded from his work that the criterion which has the desired properties is characterized by the condition that the Mach stem be normal to the flow at the transition condition. We write this as follows:

$$\delta_0 + \delta_1 = 0 = \delta_M, \tag{3}$$

where δ_M is the zero-streamline deflexion through the Mach stem. A numerical comparison of (2) and (3) is shown in figure 3. It will be seen that the new criterion is met at somewhat smaller incidence angles ω_0 than is the old one.† Equation (3) becomes a Mach-line degeneracy, i.e. $\delta_0 = 0$ and $\omega_0 = \mu_0$ (the Mach angle), at about $M_0 = 1{\cdot}48$, and it cannot be valid for M_0 smaller than this. If we understand White correctly, he has also suggested the same criterion.

Numerical data calculated from the *RR*, *MR* theory in a form suitable for comparison with experiment are presented in figures 4–9. The wave angles at the various types of confluence points were calculated by a method described by Henderson (1964), and the data include results for transition and on both sides of it. The gas was assumed to be perfect with $\gamma = \frac{7}{5}$. The transition criteria worked out from (2) and (3) are also shown.

3. Experimental work

The experiments were planned to make a detailed comparison of (2) and (3) for steady and pseudo-steady flow. It was thought necessary to do this both in the presence of and in the absence of wall boundary-layer effects because these conceivably could modify the boundary condition (1). Experiments were also planned to obtain some data on unsteady transitions. It was considered that the latter tests would provide a crucial test of the conclusion that follows from (2) and figures 2 and 3 that there will be a discontinuity in the downstream pressure during transition when $M_0 > 2{\cdot}23$. An unsteady experiment has the property that the wave system passes through the transition $RR \rightarrow MR$ during the course of the experiment and should therefore emit a band of expansion

† Note that the two criteria coincide at $M_0 = 2{\cdot}23$.

waves into the downstream flow. This band should be of finite strength and should consist approximately of cylindrical waves that begin to grow out of the reflexion point at transition. They should be detectable by a schlieren system.

The wind-tunnel models are shown in figures 1 (*a*) and (*b*). For the single wedge the reflexions occur in the presence of wall boundary layers. Numerous experiments on shock/boundary-layer interaction by Bardsley & Mair (1951), Holder & Gadd (1955), Chapman *et al.* (1958) and others have shown this to be a complicated phenomenon with the reflected wave often substantially dispersed in the near field. When there is no separation, or when there is separation followed by reattachment, it seems that the wave angle ω_1' of the reflected shock asymptotically approaches the angle predicted by inviscid perfect-gas theory. We tried to design our model (see figure 1*a*) so that the shock impinged on a thin boundary layer of high Reynolds number, so that the asymptotic value would be approached as closely as possible in the near field. We assumed that the symmetry of the other model (see figure 1*b*) would ensure that the wave system satisfied (1), and that the results would be nearly independent of boundary-layer effects.†

The pseudo-steady flows were generated in a shock tube with the models shown in figures 1 (*c*) and (*d*). A reflexion occurring on the single-corner model does so in the presence of a wall boundary layer which is itself produced by the wave system. A reflexion which occurs on the double-corner model should be nearly independent of wall boundary layers† and should also satisfy (1). The unsteady flows were generated in the same shock tube using the models shown in figures 1 (*e*) and (*f*). These are convex and concave corners formed from the surfaces of cylinders. All of the models were set up as accurately as possible, within 10′ of arc, with the help of a cathetometer.

Probably the most sensitive way of detecting the onset of Mach reflexion is by the appearance of the contact discontinuity which emerges from the wave confluence (Smith 1959); this is always visible before the Mach stem. Conversely the disappearance of this discontinuity is the most sensitive way of detecting the inverse transition. A schlieren system is therefore indicated and in fact the wave systems were photographed with a high-quality Zeiss instrument using an electric spark of duration about 1 μs. The model geometry and the wave angles were measured from the negatives with a Nikon V16 profile projector which enlarged them by a factor of about ten. This instrument is accurate to within 1′ of arc, but we could not achieve this accuracy because the shock waves were not sufficiently smooth.

Shock-tube experiments are conveniently done with fixed incident shock pressure ratios, that is with fixed shock Mach numbers M_s, but we wanted to make a direct comparison between the shock-tube and wind-tunnel results. Now the wind-tunnel work had to be done at a constant free-stream Mach number M_0, and this was initially about 2·8 but was subsequently raised to 3·0 to enhance the differences between the *RR* and *MR* systems. Accordingly the shock-

† Disturbances caused by changes in the boundary-layer displacement height can reach the wave confluence, but the effect should be small.

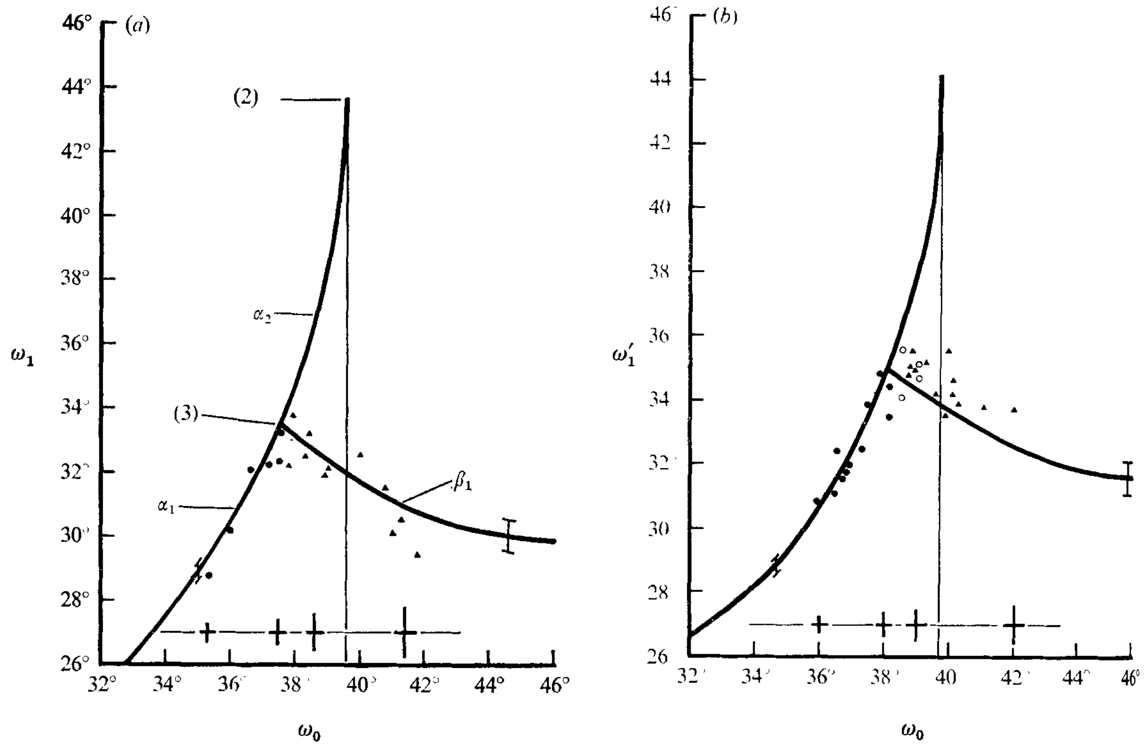


FIGURE 4. Steady-state flow. Wind-tunnel results for transition to Mach reflexion. (*a*) Single-wedge-model, $M_0 = 2{\cdot}95 \pm 0{\cdot}05$. (*b*) Double-wedge model, $M_0 = 2{\cdot}85 \pm 0{\cdot}05$. (2), transition point calculated from (2); (3), transition point calculated from (3); ●, regular reflexion, experimental point; ○, doubtful reflexion with unresolvable detail at the confluence; ▲, Mach reflexion; ——, numerical data calculated from *RR* (α_1 and α_2) and *MR* (β_1) theory; I, variation in numerical data due to variation in Mach number; +, estimated experimental error.

tube work was done so that the analogue $M_s/\sin\omega_0$ of M_0 in the pseudo-steady flows was equal to 3·0 as nearly as was practicable. The wind-tunnel data are presented in figure 4, the shock-tube data for $M_0 = 3{\cdot}0$ are presented in figure 5, with some additional data for $M_0 = 4$ in figure 6, and the unsteady data are presented in figure 7. Data on pseudo-steady and unsteady flows obtained for $M_0 = 1{\cdot}7$ are presented in figures 8 and 9 respectively. We could not get any $M_0 = 1{\cdot}7$ data on steady flows because our wind tunnel is limited to the range $2{\cdot}4 \leqslant M_0 \leqslant 3{\cdot}0$. In all of this work the estimated experimental errors are shown in figures 4–9 by error bars.

4. Discussion of the experimental results

Inspection of the results for the steady flows (figure 4) shows that the inviscid perfect-gas theory predicts the wave confluence angles accurately for both regular and Mach reflexion, and also that (3) is the correct criterion for the transition while (2) is significantly in error. The same conclusions are valid for

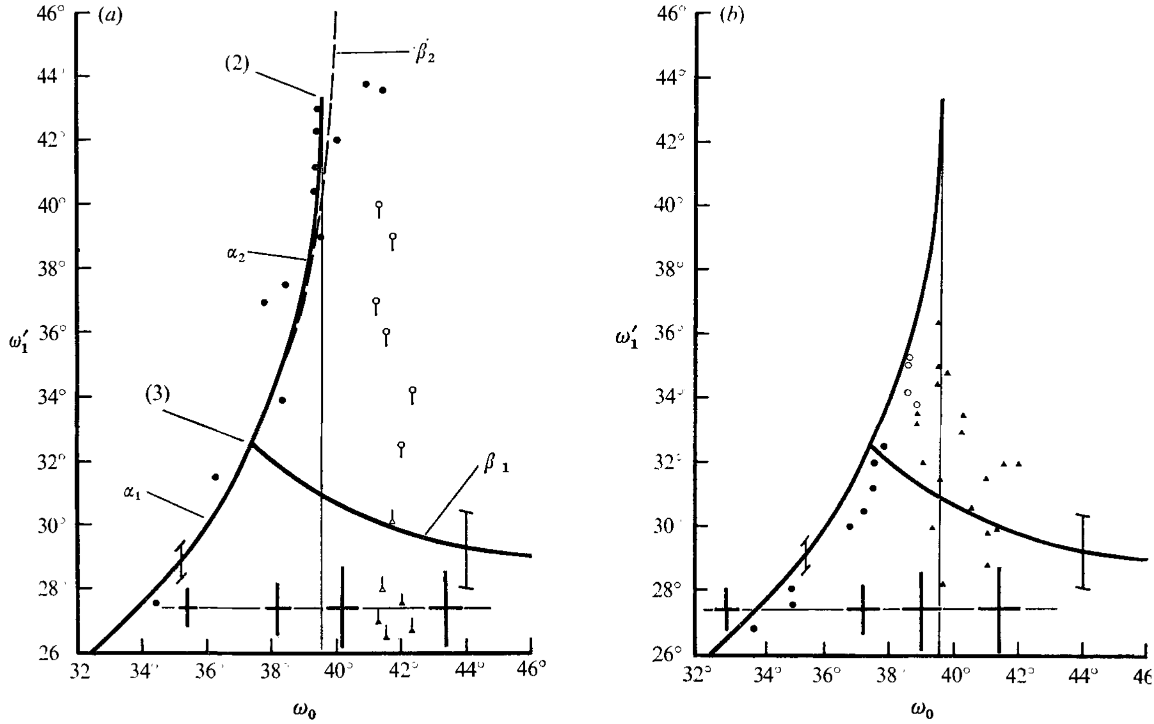


FIGURE 5. Pseudo-steady flow. Shock-tube results for transition to Mach reflexion. (*a*) Single-corner model, $M_0 = 3{\cdot}0 \pm 0{\cdot}1$. (*b*) Double-corner model, $M_0 = 3{\cdot}0 \pm 0{\cdot}1$. (2), transition point calculated from (2); (3), transition point calculated from (3); ●, regular reflexion, experimental point; ○, doubtful reflexion, with unresolved details at confluence, or Mach reflexion, with, in either case, $\omega_2' =$ reflected wave angle after kink; △, doubtful reflexion, with unresolved details at confluence, ω_1' = reflected wave angle before kink; ▲, Mach reflexion, slip line or Mach stem visible, ω_1'; φ (ω_1''), △, ▲ (ω_1'), values of wave angles for the same reflected shock wave; ——, numerical data calculated from *RR* (α_1 and α_2) and *MR* (β_1) theory; – – –, line developed in figure 10, β_2; I, variation in numerical data due to variation in Mach number, $\Delta M = \pm 0{\cdot}1$; +, estimated experimental error.

the pseudo-steady data shown in figure 5(*b*), but there is a remarkable anomaly between the results shown in figure 5(*a*) and those shown in figures 4(*a*), 4(*b*) and 5(*b*), namely that, for the figure 5(*a*) data, the regular reflexion is apparently persisting beyond the limits predicted by both (2) and (3). It therefore continues to exist in a region where the perfect-gas theory has no *RR* solution. This persistence confirms similar results obtained by Bleakney & Taub, Griffith & Bleakney and Kawamura & Saito. Nevertheless apart from this anomoly the perfect-gas theory elsewhere accurately predicts the wave angles for both regular and Mach reflexion in figure 5(*a*).

In an attempt to resolve the anomaly it was decided to make a detailed study of the polar diagram, figure 10. We began by assuming that the apparent regular reflexion that persisted beyond the limit determined by (3) was in reality a Mach reflexion, but that its Mach stem and contact discontinuity were

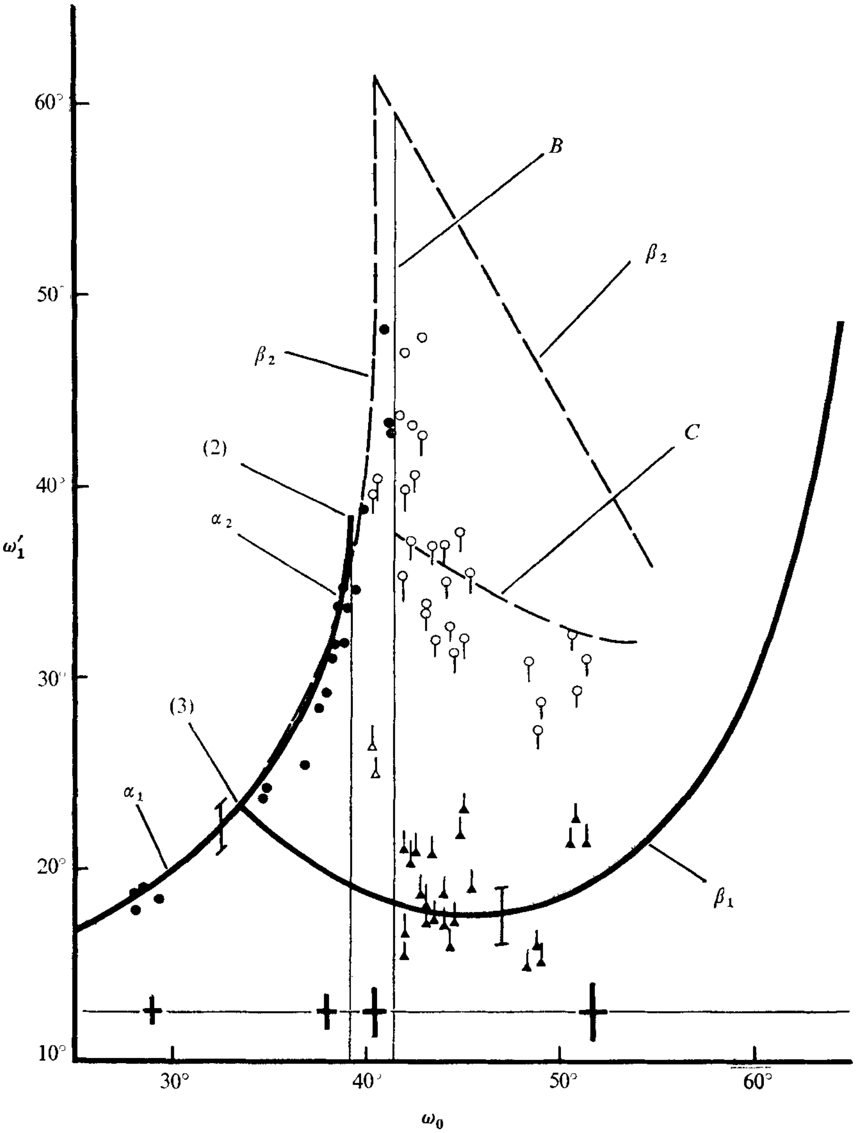


FIGURE 6. Pseudo-steady flow. Shock-tube results for transition to Mach reflexion. Single-corner model, $M_0 = 4{\cdot}0 \pm 0{\cdot}2$. For symbols see caption to figure 5. *B*, reflected wave angle for condition shown on figure 10(*e*); *C*, reflected wave angle for maximum deflexion described on figure 10(*f*).

too close to the wall to be observable. Now in figure 5(*a*), $M_0 > 2{\cdot}40$, so that the *MR* system would have to be of the double-Mach-reflexion type. If this idea is correct then what we and others have been measuring is the reflected-wave angle ω_2' at the *secondary* confluence and not the angle ω_1' at the *primary* confluence. The confluences would have to be very close together ($< 0{\cdot}1$ mm apart) for them to be unobservable, which means that steady flow theory should be valid in a neighbourhood containing both points. The rather complicated

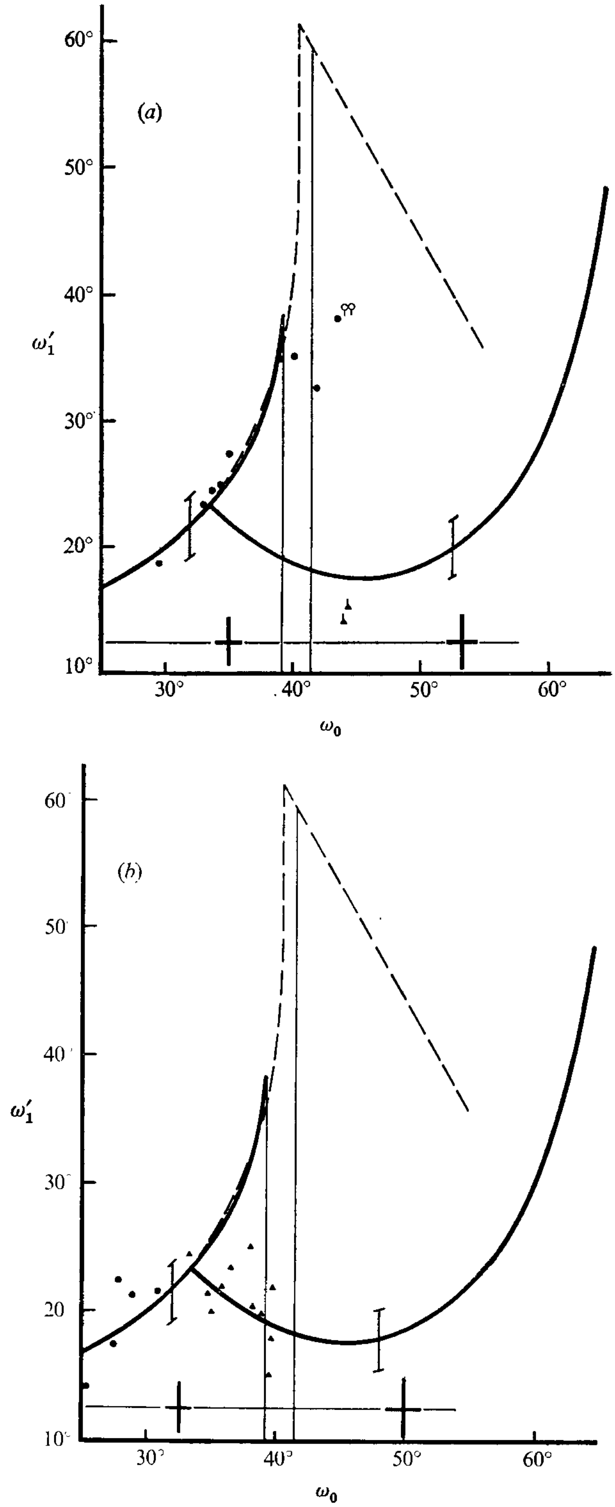


FIGURE 7. Unsteady flow. Shock-tube results for transition to Mach reflexion. (*a*) Convex-corner model, $M_0 = 4 \cdot 0 \pm 0 \cdot 3$. (*b*) Concave-corner model, $M_0 = 4 \cdot 0 \pm 0 \cdot 3$. For symbols see caption to figure 5.

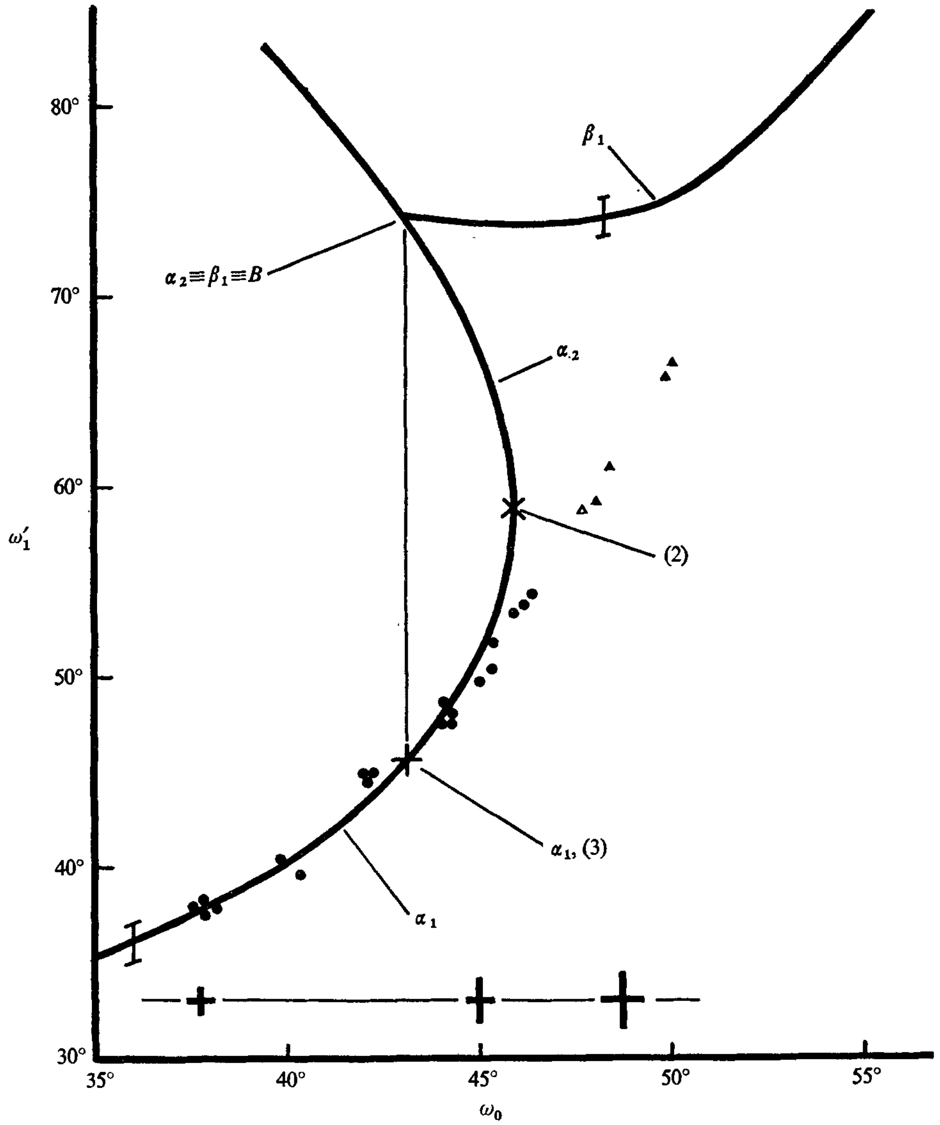


FIGURE 8. Pseudo-steady flow. Shock-tube results for transition to Mach reflexion. Single-corner model, $M_0 = 1{\cdot}7 \pm 0{\cdot}1$. $\alpha_2 \equiv \beta_1 \equiv B$, condition where (3) is met on hodograph diagram. For other symbols see caption to figure 5.

sequence of events shown in figure 10 is described in the appendix, and from it we were able to calculate the wave angle ω_2' of the reflected shock at the secondary confluence point. This is plotted as a dashed line in figure 5 (*a*), and it will be seen that for most of its length its agrees closely with the wave angle obtained from the *RR* theory, but significantly *it also extends beyond the RR theory,* which suggests that our hypothesis is correct. Further evidence is presented in figure 6, where the larger Mach number $M_0 = 4$ enhances the effects. The double Mach

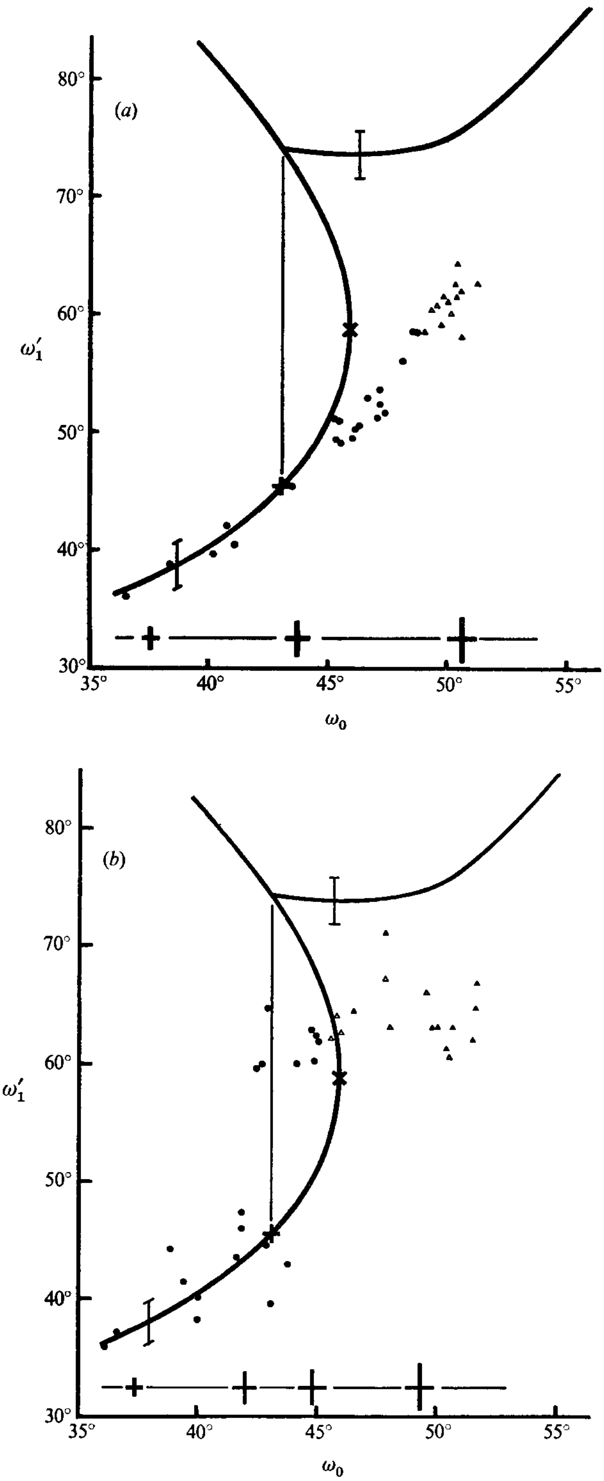


FIGURE 9. Unsteady flow. Shock-tube results for transition to Mach reflexion. (*a*) Convex-corner model, $M_0 = 1{\cdot}7 \pm 0{\cdot}2$. (*b*) Concave-corner model, $M_0 = 1{\cdot}7 \pm 0{\cdot}2$. For symbols see caption to figure 5.

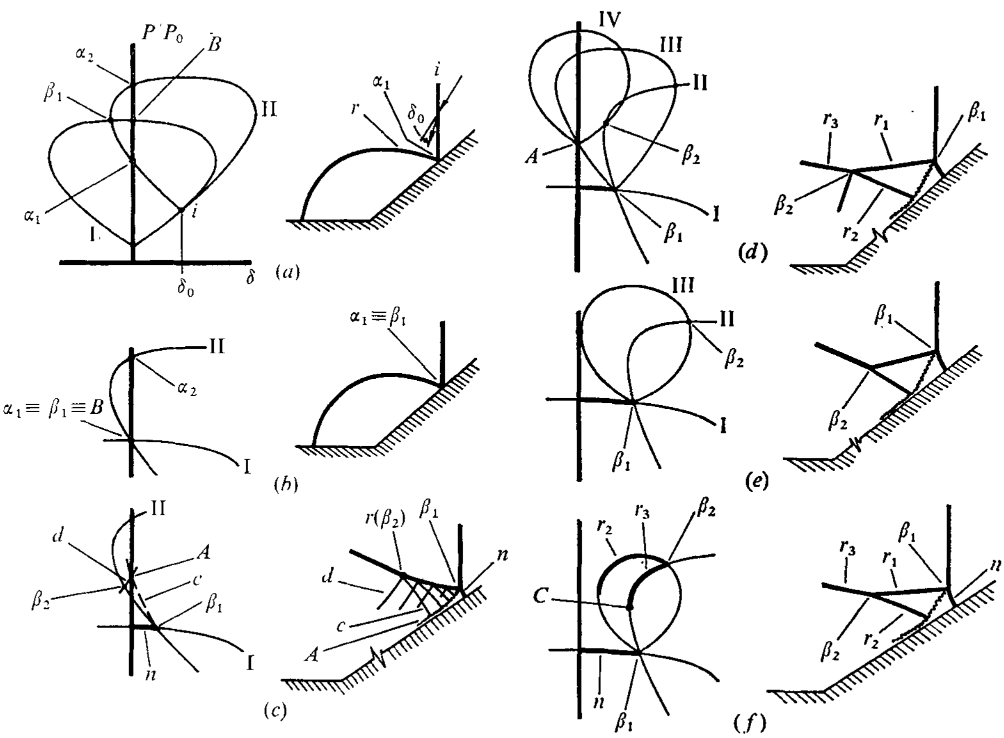


FIGURE 10. Transition to Mach reflexion with increasing incident shock wave angle, for $M_0 > 2{\cdot}40$.

reflexion becomes visible when ω_0 is sufficiently large, and it is then possible to measure the wave angles ω_1' and ω_2' of the reflected shocks at both confluences. The results of doing this are shown in figure 6. It will be seen that for the primary confluence the measured values of ω_1' are in agreement with the calculated ones β_1, while for the secondary confluence β_2 the agreement is only qualitative and is quantitatively unsatisfactory. Better agreement is obtained by using a different method for calculating ω_2' (see appendix); these curves are labelled C in figure 6. However, a still more refined theory of the β_2 confluence is needed. Although it has been impossible to demonstrate it directly, there does seem to be substantial indirect evidence to support the hypothesis that, for the data in figure 5(a), equation (3) is again the criterion for transition and that *the apparent persistence of the RR system beyond this point is really due to a double-Mach-reflexion system that has not developed sufficiently to be observable as such.*

For the unsteady data shown in figure 7, the instantaneous RR flow appears to develop from a corner at the intersection of the tangent at the point of reflexion and a normal to the flat plate at the initial corner formed by the plate and the cylinder. This normal is actually the locus of all the instantaneous corners. This property made it more difficult to control the experiments because the Mach number $M_0 = M_s/\sin\omega_0$ depended on the instantaneous incidence angle ω_0, and so the value we got depended critically on the timing of our photograph of the flow. Inevitably the data became somewhat scattered. Nonetheless the steady-state theory is surprisingly accurate and deviates beyond the limits of experimental error only occasionally. A remarkable feature of the data is that the convex-corner results have the same kind of behaviour as those shown in figures 4 (*a*), 4 (*b*) and 5 (*b*), while the concave-corner results are similar to those shown in figure 5 (*a*). Thus in the case of the concave corner we again have direct evidence that (3) is the criterion for transition, but our evidence is only indirect for the convex corner. We may also conclude that the anomalous behaviour of the data shown in figures 5 (*a*) and 7 (*a*) can be removed by replacing these boundaries with a sufficiently concave surface, but the minimum radius of curvature has not yet been determined. Significantly also, there was no sign of the emission of a band of expansion waves as the system passed through transition. For all the data presented in figures 4–7, boundary-layer and other viscous effects seem to be relatively unimportant.

The pseudo-steady data presented in figure 8 for $M_0 = 1{\cdot}7$ are for the same type of transition as is illustrated in figure 2 (*b*). The experimental data for the regular reflexion deviate increasingly from the theoretical curve as ω_0 increases, and once more they apparently extend to values of ω_0 for which the perfect-gas theory gives no solution. When the Mach reflexion becomes visible it is found that its wave angles do not agree with the three-shock theory. Similar results have been found by Bleakney & Taub, Griffith & Bleakney and Kawamura & Saito. Here also (2) fails to predict the transition, but on the other hand (3) appears to predict the *onset of the deviation* from the theoretical *RR* curve even though it apparently fails to predict the onset of Mach reflexion. We have had no success with our attempts to construct a quantitative theory that would predict the onset of Mach reflexion in the range $1{\cdot}48 \leqslant M_0 \leqslant 2{\cdot}40$, nor one that would predict the wave angles for our system that deviate from the *RR* theory. The reason may be due to the reflected shock's becoming strongly curved at the confluence point, which would make it impossible to verify a theory by measuring wave angles; but at present we do not know whether this is so.

Similar results were obtained for the unsteady data shown in figure 9. For the convex corner, (3) again predicts quite accurately the point where the experimental data begin to deviate from the *RR* theory, and the same is true for the concave corner. In both cases, (2) again fails to predict any of the data. In the Mach-number range $1{\cdot}48 \leqslant M_0 \leqslant 2{\cdot}40$, equation (2) is partly successful in that it seems to predict the onset of the deviation of the experimental data from the *RR* theory. More refined experiments will be needed to decide whether the deviation is due to shock curvature, viscous effects, or something else.

5. Conclusions

The experimental results show quite clearly that the criterion quoted in a number of standard texts, equation (2), for the transition between regular and Mach reflexion is wrong for every flow that has been investigated in detail, and these include flows which are steady, pseudo-steady and unsteady. The criterion that appears to be correct, at least for $\gamma = \frac{7}{5}$ and $M_0 \geqslant 2{\cdot}40$, is (3), and is characterized by the condition that at transition the Mach reflexion has a Mach stem that is normal to the flow. This means also that transition takes place in such a way that mechanical equilibrium of the system is preserved throughout the process. The inviscid perfect-gas theory is accurate for both regular and Mach reflexions for $M_0 \geqslant 2{\cdot}40$ in air at least up to $M_0 = 4$. This last conclusion is valid for steady and pseudo-steady flows, and is sufficiently valid to be useful even for unsteady flows. In all of these cases viscous effects seem to be of comparatively minor importance. For the range $1{\cdot}48 \leqslant M_0 \leqslant 2{\cdot}40$, the inviscid perfect-gas theory is accurate for regular reflexions up to the value of the angle of shock incidence ω_0 determined by equation (3), but beyond this limit the effects of shock-wave curvature possibly prevent an accurate comparison between theory and experiment. At $M_0 = 1{\cdot}48$, equation (3) becomes a Mach-line degeneracy, and for $M_0 \leqslant 1{\cdot}48$, it does not exist. We do not yet know what is the correct criterion for this range but on the basis of the data we have so far we would expect it would be such that the mechanical equilibrium of the system is preserved during the transition process.

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Appendix

Only a brief description of the polar diagrams is given here; further details of the technique have been given elsewhere (Guderley 1947; Henderson 1966). For $M_0 > 2{\cdot}40$ and $\gamma = \frac{7}{5}$, the sequence of events describing the transition process is illustrated in figure 10. The regular-reflexion solution is defined by the weaker (α_1) of the two points $\alpha_{1,2}$ at which the reflected shock polar II intersects the ordinate; see figure 10(*a*). The *MR* solution is defined by one of the intersection points β_1 of polars I and II. The transition condition defined by (3) is shown in figure 10(*b*), where α_1 coincides with the normal shock point B of the primary polar I ($\alpha_1 = \beta_1 = B$). For larger values of ω_0 (or δ_0), the α_1 solution continues to exist for a while, until in fact polar II becomes tangential to the ordinate ($\alpha_1 \equiv \alpha_2$), and it is this condition which corresponds to (2). However for those values of ω_0 greater than those defined by (3) we propose that the α_1 solution should be replaced by the solution points labelled $\beta_{1,2}$ in figure 10(*c*), which corresponds to a double Mach reflexion. The point β_1 is the solution for the primary confluence and the point β_2 that for the secondary confluence. The β_2 solution is constructed by plotting a characteristic curve c from β_1 until it intersects the ordinate at A. Physically the characteristic represents the band of compression waves which turn the flow back parallel to the wall after it has been deflected towards the wall by the primary Mach reflexion. Next a second characteristic d is constructed from point A, and its intersection with II defines the β_2 solution. The characteristic d represents a weak band of expansion waves which are the reflexions of the compression waves off the shock r. With further development (increasing ω_0), the β_2 solution undergoes continuous changes until the position shown in figure 10(*d*) is reached. In the process the compression fan gradually concentrates itself into a shock r_2, and the β_2 solution is now defined by the intersection of polars II and IV, where IV is constructed for a Mach number M_2

downstream of r_2. The wave angles for β_2 are shown in figures 5 and 7. The subsequent events involve some discontinuities although these appear to be of minor character. The β_2 results do not agree with sufficient accuracy for development beyond figure 10(*e*), but the wave angle calculated at the point C of maximum deflexion on II is better, and this is also shown in figure 6, but clearly the β_2 solution will need even more refined study. The theory of the β_1 confluence however now seems to be satisfactory. Photographs of the type of wave system shown in figure 10(*e*) have been obtained by Weynantes, Law & Glass, Gvozdeava *et al.* and others.

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